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$$-\frac{1+c^2}{3e}\int_{x_2}^{x_1}\frac{x^4\,dx}{\sqrt{[4ac^2(a+x)-x^2]}\{c(2a+x)+\sqrt{[4ac^2(a+x)-x^2]}\}}.$$

Let $2ac^2-x=2ac\sqrt{(1+c^2)}\cos\theta$.

$$\therefore\ V=2a^3c\int_0^\pi[c-\sqrt{(1+c^2)}\cos\theta][1+2c^2-2c\sqrt{(1+c^2)}\cos\theta]d\theta$$

$$+\tfrac{8}{3}a^3c^2\int_0^\pi[c-\sqrt{(1+c^2)}\cos\theta]^3d\theta-\tfrac{8}{3}a^3c^2\sqrt{(1+c^2)}\int_0^\pi[c-\sqrt{(1+c^2)}\cos\theta]^2\sin\theta d\theta$$

$$-\tfrac{8}{3}a^3c^2\sqrt{(1+c^2)}\int_0^\pi\frac{[c-\sqrt{(1+c^2)}\cos\theta]^4d\theta}{\sin\theta-c\cos\theta+\sqrt{(1+c^2)}}.$$

$$\therefore\ V=\frac{2\pi a^3c^2}{3}(6+15c^2+10c^4)-\tfrac{16}{9}a^3c^2\sqrt{(1+c^2)}(1+4c^2)$$

$$-\tfrac{8}{3}a^3c^2\sqrt{(1+c^2)}\int_0^\pi\frac{[c-\sqrt{(1+c^2)}\cos\theta]^4d\theta}{\sin\theta-c\cos\theta+\sqrt{(1+c^2)}}.$$

Let $c=\cot\beta$.

$$\therefore\int_0^\pi\frac{[c-\sqrt{(1+c^2)}\cos\theta]^4d\theta}{\sin\theta-c\cos\theta+\sqrt{(1+c^2)}}=\frac{1}{\sin^3\beta}\int_0^\pi\frac{(\cos\beta-\cos\theta)^4d\theta}{1-\cos(\theta+\beta)}$$

$$=\frac{8}{\sin^3\beta}\int_0^\pi\sin^2\tfrac{1}{2}(\theta+\beta)\sin^4\tfrac{1}{2}(\theta-\beta)d\theta=D.$$

Let $\theta-\beta=2\varphi$. $\therefore\ \theta+\beta=2\varphi+5\beta$.

$$\therefore\ D=\frac{16}{\sin^3\beta}\int_{-\frac{1}{2}\beta}^{\frac{1}{2}(\pi-\beta)}\sin^2(\varphi+\beta)\sin^4\varphi d\varphi$$

$$=\frac{16}{\sin^3\beta}\int_{-\frac{1}{2}\beta}^{\frac{1}{2}(\pi-\beta)}(\sin^6\varphi\cos^2\beta+2\sin\beta\cos\beta\sin^5\varphi\cos\varphi+\sin^4\varphi\cos^2\varphi\sin^2\beta)d\varphi$$

$$=\tfrac{1}{2}\pi\operatorname{cosec}\beta(1+5\cot^2\beta)-\tfrac{2}{3}(1+4\cot^2\beta)=\tfrac{1}{2}\pi\sqrt{(1+c^2)}(1+5c^2)-\tfrac{2}{3}(1+4c^2).$$

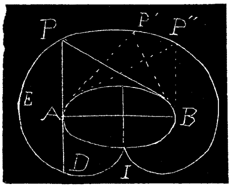
$$\therefore\ V=\tfrac{2}{3}\pi a^3c^2(4+3c^2).$$

103. Proposed by HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.

A park, in the shape of an ellipse whose diameters are 100 and 50 rods, respectively, is surrounded by a wall: one end of a rope, whose length is the circumference of the ellipse, is fastened (outside of the wall) at one end of the longer diameter and the other end at the other end of the same diameter. Over how much surface will a horse graze, which is fastened to a ring moving freely on the rope ?

Remarks by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

No correct solution of this problem has yet been received. The problem has been misinterpreted by several of our contributors, who have mistaken it as a corollary of Graves' Theorem. This was the interpretation according to which Dr. Haskell made his solution and which was inadvertently published in the MONTHLY, Vol. V, No. 4, page 111.



The problem as proposed is quite different and is not easy to solve, perhaps impossible.

The area grazed over consists of three parts :

(1) The part $ABP''P$—the area of this part is not difficult to find as the arc $PP'P''$ is the arc of a confocal ellipse; (2) two times the part $PEDAP$,—this part is generated by point, P, constrained to move under the composition of circular and elliptic evolutary motion ; and (3) two times the part $ADIA$, which is between the evolute of the elliptic field, the field, and the radius vector AD.

The part $PEDAP$ is the difficult part of the area to compute.

If any of our contributors will furnish a correct and complete solution of this problem it will be published in the next issue of the MONTHLY.

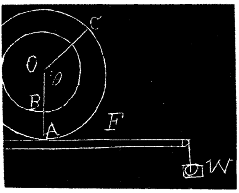
MECHANICS.

98. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

A spool, with light thread wound around, is placed upon a rough table so that the thread will emerge from beneath the spool. The thread is passed over a smooth pulley at end of table and a weight attached, the pulley being so adjusted that thread is parallel to surface of table. If friction between spool and table is sufficient to prevent slipping, determine motion of spool and weight. [From problems in Mechanics at Harvard University.]

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let T=tension on thread, x=distance spool's center moves, y=distance weight W moves, θ=angle AOC through which spool turns, $OA=a$, OB the radius of the thread=b.



Then $x=a\theta$, $y=(a-b)\theta$.

The forces acting on the spool are friction, F; tension, T; and reaction, R, perpendicular to the table.

Let m=mass of spool and thread, m_1=mass of weight. Then

$$m\frac{d^2x}{dt^2}=T-F,\ mk^2\frac{d^2\theta}{dt^2}=Fa\,..(1,2),\text{ for spool};\ m_1\frac{d^2y}{dt^2}=T-m_1g\,..(3),\text{ for weight.}$$

Eliminating F between (1, 2) we get

$$ma\frac{d^2x}{dt^2}+mk^2\frac{d^2\theta}{dt^2}=T\,....(4)\ ;\text{ but }\frac{d^2x}{dt^2}=a\frac{d^2\theta}{dt^2},\ \frac{d^2y}{dt^2}=(a-b)\frac{d^2\theta}{dt^2}.$$